#### Ashdon Primary School Mathematical Calculation Strategies

This document provides an overview of the main strategies that the children will use when working in mathematics in our school. Concrete strategies are ones that use physical equipment, pictorial strategies use drawings and abstract strategies use numbers and symbols. The teacher will decide the most suitable strategies to be used by each pupil, dependent on their mathematical understanding and the support they require. The strategies outlined are not exhaustive, so other methods may also be used in addition to those within this document.

## Addition

Strategy	Concrete	Pictorial	Abstract
Combining two	Use cubes to add two amounts together as a	Use pictures to add two amounts together as a	Use the part-whole diagram below to move
parts to make a	group or as a bar.	group or bar.	into the abstract representation.
whole: part- whole model.	Use Numicon to add to groups together as a group.	3       0	7 8 7 + 8 = 15 15 = 7 + 8
Starting at the	Start with the larger number on a bead string	Start at the larger number on the number line	Place the larger number in your head and
bigger number	or number track and then count on the smaller	or number track and count on in ones, or in	count on the smaller amount to find the total.
and counting on.	amount, 1 at a time to find the answer.	one jump, to reach the total.	
	1. CD00000000 ( D00000000 )	3+4=	5 + 12 = 17 would be calculated 12 + 5.
	My 1 to 10 1 2 5 6 7 8 9 10	3 + 5 = 0 + 2 + 3 + 5 + 5 + 7 + 8 + 9 + 10	

Strategy	Concrete	Pictorial	Abstract
Regrouping to	Start with the larger number and use the	Use pictures or a number line. Regroup or	7 + 4 = 11
make 10.	smaller number to make 10.	partition the smaller number to make 10.	
	9 + 3 =		If I am at seven, how many more do I need to make 10? How many more do I have left to add on now?
	6 + 5 =	3 + 9 =	
		9 + 5 = 14 $+1$ $+4$ $+1$ $+1$ $+4$ $+1$ $+1$ $+4$ $+1$ $+1$ $+4$ $+1$ $+1$ $+4$ $+1$ $+1$ $+4$ $+1$ $+1$ $+4$ $+1$ $+1$ $+1$ $+1$ $+1$ $+1$ $+1$ $+1$	
Adding three	Make 10 with 2 of the 3 amounts, if possible,	Add together 3 groups of drawn objects. Draw	Combine two numbers to make 10 and then
single digit	then add the third amount.	a picture to recombine the groups to make 10	add the remaining amount.
numbers		and add the remaining amount.	
Destitioning	4 + 7 + 6 = 17 Add the 4 and 6 together to make 10, then add on the 7. 3 + 6 + 7 = 16 or 3 + 6 + 7 = 16	$ \begin{array}{rcl}  & & & & & & & & & & & & & & & & & & &$	4 + 7 + 6 = 10 + 7 = 17
Partitioning	Use Base 10 (Dienes) to regroup the 10's	34 + 2 5 = 59	
	Tens Ones	111 + 11 = 11111 +	$ \begin{array}{c} 45 + 33 \\ = 40 + 5 + 30 + 3 \\ = 70 + 8 \\ = 78 \end{array} $ $ \begin{array}{c} 245 + 633 \\ = 800 + 70 + 8 \\ = 878 \end{array} $

Strategy	Concrete	Pictorial	Abstract
Column method –	Add together the ones first, then add the tens	After practically using the Base 10 (Dienes)	Start by partitioning the numbers in an
no regrouping	using Base 10 (Dienes). For example, 34 + 15 =	equipment, children can draw the amounts to	expanded method, arranged like the example
	Set out the amounts in columns first	help them solve additions.	below: 20 + 1 40 + 2 60 + 3 = 63
	Combine the ones, then the tens		Progress on to the compacted method: T 1's 2 1 $\frac{+4 2}{6 3}$
Column mothod	Make both numbers using Pase 10 (Dispas)	Children can draw a nictorial representation of	
Column method -	make both numbers using Base 10 (Dienes)	the columns and Base 10 (Dienes) equipment to	expanded method, arranged like the example:
regrouping	on a place value grid.         Image: Construction of the second	the columns and Base 10 (Dienes) equipment to further support their learning and understanding.	expanded method, arranged like the example: 20 + 5 $40 + 8$ $60 + 13 = 73$ before moving on to clearly showing the exchange in a compacted column method. H T 1's 5 3 6 + 8 5 $\frac{11}{621}$ The children will then move on to using the column method to work out totals involving decimals, initially in the context of money, such as: $f 2 3 . 5 9$ $+ f 7 . 5 5$ $\frac{1 1 1}{f 3 1 . 1 4}$

### **Subtraction**

Strategy	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters or cubes, etc. to show how objects can be taken away, one at a time.	Cross out drawn objects, to show what has been subtracted and how many remain.	18 – 3 = 15 8 – 2 = 6
	5-3=2	14 - 6 = 8	
Counting back	Make the larger number in your subtraction using beads on a bead string or cubes joined together. Count backwards, separating the subtracted amount from the remainder. 13 - 4 = 9	Count back on a number line or number track. Start at the bigger number and count back the smaller number, showing jumps on the number line. 9 10 11 12 13 14 15 Progress to counting back in steps of 10. -10 $-10$ $-1$	Put an amount in your head and count backwards, initially in ones, then count back in 10's, then subtract the remaining ones. For example, put 13 in your head and count back 4. What number are you at? Use your fingers to help. Or 53 - 35 = -10 -10 -10, -1, -1, -1, -1, -1 43, 33, 23, 22, 21, 20, 19, 18
	10 - 4 = 6 1 2 3 4 5 6 7 8 9 10		



Strategy	Concrete	Pictorial	Abstract
Make 10	Use ten frames, with counters or cubes, to take an amount so that only 10 are left, then subtract the remaining number required. For example, 14-6=8	Use a number line and subtract an amount to reach 10, then take away the rest. For example, 14-6=8	How many do we have to take off to reach the next 10? How many do we have left to take off? For example, 16 – 8 = Take off 6 to get to 10, then 2 more to subtract.
Partitioning	Use Base 10 (Dienes) to regroup the 10's together and the ones together. Remove the 10s and 1's with your hands to see what's left.	Draw the Base 10 (Dienes) equipment, crossing out the items subtracted. For example, 37 – 24 = 13	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
Column method without regrouping	Use Base 10 (Dienes) equipment to make the larger number and then take away the smaller number. 54 – 22 = 32 Start with	Draw the Base 10 (Dienes) alongside the written calculation to help show working. T 1's T 1's 5 4 -2 2 3 2	Initially use expanded form, such as the example below $(47 - 23)$ : 40 7 <u>-20 4</u> <u>20 + 3</u> = 23 This will lead to a clear written column subtraction. T 1's 3 2 <u>-1 2</u> <u>2 0</u>

Concrete	Pictorial	Abstract
Using Base 10 (Dienes) equipment, make the	Draw the Base 10 (Dienes) alongside the	Children can start their formal written method
larger number, then subtract the ones,	written calculation to help show working.	by partitioning the number into clear place
velocities and the tens, exchanging (swapping)	exchanges are represented by the green	value columns.
Initially complete questions with one		<sup>700</sup> <del>800</del> <sup>1</sup> 30 6 <u>- 200 50 4</u> <u>- 202 50 2</u>
exchange required before moving on to	lens Ones 5_1 65	<u>500 80 2 = 582</u>
(Green 10 used to highlight the exchange for	$\frac{-28}{37}$	Moving forward, the children can use a more compact method.
clarity in these photos)		Ц Т 1/c
65 – 28 =		<sup>6</sup> 7 <sup>1</sup> 2 8
Make initial amount         Image: Image of the second secon	HundredsTensOnes $435$ $-273$ $262$	$\frac{-582}{146}$ This will lead using the same method using decimals. $512 1$ $2.63 . 0$ $-\frac{26.5}{236.5}$
	Concrete         Using Base 10 (Dienes) equipment, make the larger number, then subtract the ones, followed by the tens, exchanging (swapping) where required.         Initially complete questions with one exchange required before moving on to subtractions with 2 or more exchanges.         (Green 10 used to highlight the exchange for clarity in these photos)         65 – 28 =         Make initial amount         Exchange a 10 for 10 ones         Subtract the desired amount	ConcretePictorialUsing Base 10 (Dienes) equipment, make the larger number, then subtract the ones, followed by the tens, exchanging (swapping) where required.Draw the Base 10 (Dienes) alongside the written calculation to help show working. Exchanges are represented by the green markings.Initially complete questions with one exchange required before moving on to subtractions with 2 or more exchanges. (Green 10 used to highlight the exchange for clarity in these photos)Image in the exchange for clarity in these photos)65 - 28 = Make initial amountImage in the exchange in the exchange in the exchange in the exchange a 10 for 10 onesImage in the exchange

# Multiplication

Strategy	Concrete	Pictorial	Abstract
Repeated	Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits	Write addition sentences to describe objects
addition		on it. How many biscuits are there altogether?	and pictures.
	3+3+3	$\begin{array}{c} \swarrow & \bigstar \\ & \swarrow & \swarrow & \swarrow \\ & & 2 \text{ add } 2 \text{ add } 2 \text{ equals } 6 \end{array} \qquad \qquad$	
		5 5 5 5 5 5 5 5 5 5 5 5 5 5	2+2+2+2=10
		Bar model can be used:	
		3 3 3 3 3 3 3	
Doubling	Use practical activities to show how to double	Draw pictures to show how to double a	Partition a number and then double each part
	a number.	number. Double 4 is 8	before recombining it back together.
	double 4 is 3 4×2 = 8		$ \begin{array}{c} 10 \\ 10 \\ 1 \\ x_2 \\ 20 \\ 12 \end{array} $
Counting in	Count in multiples, supported by real objects	Use a number line to continue to support in	Count in multiples of a number aloud.
multiples	in equal groups.	counting in multiples.	
			Write sequences with multiples of numbers. For example,
		0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 $4 \times 5 = 20$	2, 4, 6, 8, 10
	Use physical objects, e.g. counters, to move in		5, 10, 15, 20, 25, 30, 35, 40
	equal steps along a number track.		
	Use Numicon to count in multiples.		

Strategy	Concrete	Pictorial		Abstract		
Arrays – showing	Create arrays using counters / cubes to show	Draw arrays in different rotations to find	Use an array to w	rite multipl	ication se	ntences
commutative	multiplication sentences.	commutative multiplication sentences.	and reinforce rep	eated addit	ion.	
multiplication		4×2=8 2×4=8 0 0 4×2=8 0 4×2=8		000	0	
		Link arrays to areas of rectangles.	5 + 5 + 5 = 15			
			3 + 3 + 3 + 3 + 3 = 5 x 3 = 15 3 x 5 = 15	15		
Grid method	Arrange Base 10 equipment as a grid, such as	They can draw the Base 10 (Dienes)	Start with multipl	ying by one	-digit nun	nbers
	the example below:	equipment on the grid.	and showing the grid.	lear additio	on alongsi	de the
	20 2		For example, 35 x	7 = 210 + 3	5 = 245	
			Х	30		5
			7 Moving forward,	210 nultiply by	a 2-digit r	35 number.
			13 x 18 = 100 + 80	+ 30 + 24 =	= 234	
			X	10		8
	30-		10	100		80
			3	30		24
			1342 x 18 = 10000+3000+800 24,156	0+2400+40	0+320+20	+16 =
			X 1000	300	40	2
				3000	400	20
				2400	320	10

Strategy	Concrete	Pictorial	Abstract
Column		24 x 3 = 72	Write the calculation fully expanded:
multiplication		(Note that the green marking illustrates where	T 1's
		an exchange has taken place)	32
			<u>X 24</u>
		Hundreds Tens Ones	8 (2 x 4)
			120 (30 x 4)
			40 (2 x 20)
			<u></u>
			This calculation could be compacted slightly,
			similar to the example below:
			T 1's
			32
			$\frac{X-24}{120}$
			$128 (32 \times 4)$
			<u> </u>
			Moving on to examples of questions which
			require exchange:
			Th H T 1's
			1342
			<u>x 18</u>
			10736
			<u>13420</u>
			24156

### Division

Strategy	Concrete	Pictorial	Abstract
Halving	Use physical equipment and share into 2 equal	Children use pictures or shapes to share	Share 14 buns between two people.
	groups.	quantities. For example, 10 ÷ 2 = 5	
			14 ÷ 2 = 7
Grouping	Divide quantities of objects into equal groups.	Draw overall amount into equal groups of	28 ÷ 7 = 4
		objects.	Divide 28 into 7 groups. How many are in each group?
Division as repeated	Use number beads to display the total number, then subtract repeatedly until they	Use a number line to show jumps in equal steps.	
subtraction	reach 0. For example, 20 ÷ 5 =		
	-22222 -22222 -22222 -22222 -22222 -		
	Create a tower, or bar, using cubes joined together. Break of the same amount repeatedly until all cubes have been grouped.	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	

Strategy	Concrete	Pictorial	Abstract
Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.	Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences.
	For example, $5 \times 3 = 15$ $15 \div 3 = 5$ $3 \times 5 = 15$ $15 \div 5 = 3$		7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	Divide objects between groups and see how much is left over.	Draw dots and group them to divide an amount and clearly show a remainder.	Complete written divisions and show the remainder using r.
	For example, 14 ÷ 3 =	Use a number line to jump in equal amounts, then see how many more you need to jump to find a remainder. For example, $17 \div 5 = 3 r 2$	For example, 29 ÷ 8 = 3 r 5.

Strategy	Concrete	Pictorial	Abstract
Chunking		The children can continue to use drawn	Without remainder:
		diagrams with dots or circles to help them	155
		divide numbers into equal groups.	<u>- 50</u> (10 × 5)
		Pupils will be encouraged to move towards counting in multiples to divide more efficiently.	$\begin{array}{c} 105 \\ -50 \\ 55 \\ -50 \\ -50 \end{array}  (10 \times 5) \\ (10 \times 5) \\ -50 \\ $
			$\frac{-5}{0} \qquad (1 \times 5)$
		This also links to repeated subtraction on a number line. For example, 300 ÷ 5 = 60	Therefore 155 ÷ 5 = 31 With remainder:
		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	73 ÷ 5 5 73 -50 (10 × 5) 23 10 + 4 = 14 -20 3 (4 × 5) How many 5s have been subtracted? 14 sets of 5, with 3 left over. Answer: 73 ÷ 5 = 14 r3
Short division			Begin with divisions the divide equally, with no remainder:
			$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
			Progress on to divisions with a remainder:
			$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
			Apply this to decimal numbers (remainder might also be represented as a fraction):
			$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

Strategy	Concrete	Pictorial	Abstract
Long division			How many groups of 12 thousands do we
			have? None. Exchange 2 thousand for 20
			hundreds.
			0
			12 2544
			How many groups of 12 are in 25 hundreds? 2 groups. Circle them
			We have grouned 24 hundreds so can take
			them off and we are left with one
			12 2544
			24
			1
			Exchange the one hundred for ten tens so now
			we have 14 tens. How many groups of 12 are
			in 14? 1 remainder 2
			24
			14
			12
			2
			Exchange the two tens for twenty ones so now
			we have 24 ones. How many groups of 12 are
			0212
			$12 \sqrt{2^2 \Gamma^2}$
			12  2344
			14
			24
			24
			0